lecture bases consolidation

**Introduction and conversion from a base to base 10, and hexadecimal digits**

1. We work, generally, in decimal also called base 10. There are 10 digits 0, 1 ,2, 3, 4, 5, 6, 7, 8, 9. When counting if we get to 9 in a column we then go back to 0 and add 1 to the next column so 18, 19, 20 and 198, 199, 200
   1. Binary is base 2. There are 2 binary digits 0, 1. When counting if we get to 1 in a column we then go back to 0 and add 1 to the next column so 1, 10 and 100, 101, 110
   2. Octal is base 8. There are 8 octal digits 0, 1 ,2, 3, 4, 5, 6, 7. When counting if we get to 7 in a column we then go back to 0 and add 1 to the next column so 16, 17, 20 and 176, 177, 200
2. A subscript is used to identify the base of a number:
   1. is octal
   2. is base 2
   3. is base 7
   4. is decimal but is more often written unless there is a need to avoid confusion
3. In decimal the number has column values or this can also be written

So the number 426 is made of

1. Counting in binary

|  |  |
| --- | --- |
| decimal | binary |
| 0 | 0 |
| 1 | 1 |
| 2 | 10 |
| 3 | 11 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |
| 8 | 1000 |
| 9 | 1001 |
| 10 | 1010 |
| 11 | 1011 |
| 12 | 1100 |
| 13 | 1101 |
| 14 | 1110 |
| 15 | 1111 |
| 16 | 10000 |

Notice that

1. Look at 101 in base 2, it is made up of 100+1 which is 4+1

we can put column headers, as we did for the decimal number above

So in decimal

We can use this to convert from binary to decimal

To convert to decimal start by putting column headers

(these can also be written as powers of 2)

The binary number

1. This is called expanded form. In expanded form
2. hexadecimal is base 16

There are 16 hexadecimal digits 0, 1 ,2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. When counting if we get to F in a column we then go back to 0 and add 1 to the next column so 1E, 1F, 20 and 1FE, 1FF, 200

1. The following table converts between decimal and other bases

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| decimal | binary | octal | hexadecimal | base 5 |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 10 | 2 | 2 | 2 |
| 3 | 11 | 3 | 3 | 3 |
| 4 | 100 | 4 | 4 | 4 |
| 5 | 101 | 5 | 5 | 10 |
| 6 | 110 | 6 | 6 | 11 |
| 7 | 111 | 7 | 7 | 12 |
| 8 | 1000 | 10 | 8 | 13 |
| 9 | 1001 | 11 | 9 | 14 |
| 10 | 1010 | 12 | A | 20 |
| 11 | 1011 | 13 | B | 21 |
| 12 | 1100 | 14 | C | 22 |
| 13 | 1101 | 15 | D | 23 |
| 14 | 1110 | 16 | E | 24 |
| 15 | 1111 | 17 | F | 30 |

1. The largest 4 ‘digit’ number in base 8 is 7777

So the largest decimal number that can be written with 4 base 8 digits is

1. If you can write the numbers 0, 1, 2…4095 with 4 base 8 digits, this will be a total of 4096 different numbers that you can write with 4 base 8 digits

**Converting from decimal, base 10, to another base**

1. Calculating remainders: share 32 sweets between 5 people, each will get 6 and there will be 2 remaining. This can be written:
2. Converting from decimal to a base

convert to base 5. Because you are converting to base 5 you must divide by 5

(r is an abbreviation for remainder)

the answer is carried down each time

do not forget the last step

The answer is given by reading the remainders upwards 3310

So

1. If you are converting to hexadecimal remember to write to remainders as hexadecimal digits
2. Another way to convert from decimal to a base:

Convert to base 5

Find the largest power of 5 that goes into 215:

Powers of 5: 1, 5, 25, 125, 625…625 is too large.

Prepare column headers from 125 down to 1

So how many times does 125 go into 215? 1 put this is the 125 column. You are left with

Then the next smaller power: how many times does 25 go into 90? 3 put this in the 25 column. You are left with

How many times does 5 go into 15? 3 put this in the 5 column. You are left with

How many times does 1 go into 0? 0 put this in the 1 column

So

**Conversion between binary and octal or hexadecimal**

It is often useful to use ‘leading’ 0s in the table in question 5.

Complete the following table using 3 bit and 4 bit binary numbers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| decimal | 3 bit binary | octal | 4 bit binary | hexadecimal |
| 0 | 000 | 0 | 0000 | 0 |
| 1 | 001 | 1 | 0001 | 1 |
| 2 | 010 | 2 | 0010 | 2 |
| 3 | 011 | 3 | 0011 | 3 |
| 4 | 100 | 4 | 0100 | 4 |
| 5 | 101 | 5 | 0101 | 5 |
| 6 | 110 | 6 | 0110 | 6 |
| 7 | 111 | 7 | 0111 | 7 |
| 8 | Not possible | 10 | 1000 | 8 |
| 9 | Not possible | 11 | 1001 | 9 |
| 10 | Not possible | 12 | 1010 | A |
| 11 | Not possible | 13 | 1011 | B |
| 12 | Not possible | 14 | 1100 | C |
| 13 | Not possible | 15 | 1101 | D |
| 14 | Not possible | 16 | 1110 | E |
| 15 | Not possible | 17 | 1111 | F |

1. The table above can be used to convert 3 bit binary numbers to octal:

0

And 4 bit binary numbers to hexadecimal:

1. Add leading zeros as needed before using the table above to convert binary numbers to octal or hexadecimal
2. When converting longer binary numbers to base 8, they should be split into groups of 3 from the right and any leading zeros added if necessary. For example should be written as follows: .
3. Then each 3 bit binary number is converted into its octal equivalent so is split to . The conversion is then written underneath like this: This completes the conversion so
4. When converting longer binary numbers to base 16, they should be split into groups of 4 from the right and any leading zeros added if necessary. For example should be written as follows: . Once a binary number is split as shown into groups of 4 bits each 4-bit binary number is converted into its hexadecimal equivalent so is split to . The conversion is then written underneath like this: This completes the conversion so
5. To convert octal numbers to binary each octal digit should be converted to a 3-bit binary number so
6. To convert hexadecimal numbers to binary each hexadecimal digit should be converted to a 4-bit binary number so